A.P

① $T_n = a+(n-1)\times d$

② $S_n = \frac{n}{2}\{2a+(n-1)\times d\}$

③ $S_n = \frac{n}{2}(a+l)$

(i) Three no. in A.P & their sum is given then the no. is

$$(a-b),\ a,\ (a+d)$$

(ii) Four no.

$$(a-3b),\ (a-b),\ (a+b),\ (a+3b)$$

(iii) For five no.

$$(a-2b),\ (a-b),\ a,\ (a+b),\ (a+2b).$$

Arithmetic Progression (A.P.)

The series is arranged in a systematic way in assembly ascending or descending order. It called A.P.

Q1 Find the 10th term of the AP.

3, 8, 13, 18, ...

⇒ $a = 3$, $n = 10$

$d = a_2 - a_1 = (8-3) = 5$

$= a_2 - a_1 = (13-8) = 5$

$T_n = a + (n-1) \times d$

$T_{10} = 3 + (10-1) \times 5$ [Putting n = 10 ...]

$= 3 + 9 \times 5 = 3 + 45 = 48$

$\therefore T_{10} = 48$

Q2 How many terms are there in AP.

20, 25, 30, ... 100

$T_n = 100$

Given, $a = 20$, n^{th} term $= 100$, $d = a_1 - a$

$d = a_1 - a = 25 - 20 = 5$

$T_{nth} = a + \ldots$

$100 = 20 + (n-1) \times d$ $5 = 100 - 20 = (n-1) \times d$ 5

$= 20 + \frac{80}{5} = n - 1 \times d$ $= 16 + 1 = n$

$n = 17$

Type II

④ Find the 10th term of the sequence 10, 5, 0, -5, -10, ...

⇒ Given, $a = 10$, $T_n = 10$ and
$d = (5-10)/(0-5) = -5$.

$T_n = a + (n-1)d$

$T_{10} = 10 + (10-1) \times (-5)$

$= 10 + 9 \times (-5)$

$= 10 - 45 = -35$

∴ 10th term of the sequence is -35.

⑤ Find the 10th term of the sequence
3, 5, 7, 9, ...

⇒ Given, $a = 3$, $n = 10$ and
$d = (5-3)/(7-5) = 2$

$T_n = a + (n-1)d$

$T_{10} = 3 + (10-1) \times (2)$

$= 3 + 9 \times (2)$

$= 3 + 18 = 21$

∴ 10th term of the sequence is 21.

(20) The sum of three no. in A.P. is -24 and their product is 288. Find the number.

⇒ Let the three no.s be $\alpha-\beta$, α, $\alpha+\beta$.

A/Q,

$\Rightarrow \alpha-\beta+\alpha+\alpha+\beta=-24$

$\Rightarrow 3\alpha=-24$

$\Rightarrow \alpha=\frac{-24}{3}=-8.$

Again A/Q,

$\Rightarrow (\alpha-\beta)\times\alpha\times(\alpha+\beta)=288$

$\Rightarrow (\alpha-\beta)(\alpha+\beta)\alpha=288$

$\Rightarrow (\alpha^2-\beta^2)\alpha=288$

$\Rightarrow \alpha^3-\alpha\beta^2=288$ —— eq.

Now, putting the value of α in eq.

$\Rightarrow (-8)^2-(-8)\beta^2=288$

$\Rightarrow -512+8\beta^2=288$

$\Rightarrow 8\beta^2=288+512$

$\Rightarrow \beta^2=\frac{800}{8}=100 \Rightarrow \beta=\sqrt{100}=10$

$\therefore \alpha-\beta=-8-10=-18$

$\alpha=-8$ and $\alpha+\beta=-8+10=2.$

(21) The sum of three no in A.P is 15 and the product of ~~last and first~~ first and last is 21. Find the numbers.

⇒ Let the three no.s be $\alpha-\beta, \alpha, \alpha+\beta$

A/Q,

$\Rightarrow \alpha-\beta+\alpha+\alpha+\beta = 15$

$\Rightarrow 3\alpha = 15$

$\Rightarrow \alpha = \frac{15}{3} = 5$

Again A/Q,

$\Rightarrow (\alpha-\beta)(\alpha+\beta) = 21$

$\Rightarrow \alpha^2 - \beta^2 = 21$

$\Rightarrow (5)^2 - \beta^2 = 21$

$\Rightarrow 25 - \beta^2 = 21$

$\Rightarrow 25 - 21 = \beta^2$

$\Rightarrow 4 = \beta^2$

$\Rightarrow \beta = \sqrt{4} = 2.$

Q3 Find the A.P whose 7th and 13th terms are 34 and 64 respectively.

$T_n = a + (n-1)d$

$T_7 = a + (7-1) \times d = 34$

$= a + 6d = 34$ ——— (i)

$T_n = a + (n-1)d$

$T_{13} = a + (13-1) \times d = 64$

$= a + 12d = 64$ ——— (ii)

Subtracting (i) from (ii)

$$\begin{array}{r} a + 12d = 64 \\ a + 6d = 34 \\ \hline 6d = 30 \end{array}$$

$\Rightarrow d = 30/6 = 5$

$\Rightarrow$ Putting $d = 5$ in eq. (i)

$\Rightarrow a + 6 \times 5 = 34 \Rightarrow a = 34 - 30 = 4$

So, for this A.P first term $(a) = 4$ &

Common differ. $(d) = 5$

So the AP is 4, 9, 14, 19, ...